

Anomalous dimension of non-singlet Wilson
operators at $O(1/N_f)$ in deep inelastic scattering.

J.A. Gracey,
Department of Applied Mathematics and Theoretical Physics,
University of Liverpool,
P.O. Box 147,
Liverpool,
L69 3BX,
United Kingdom.

Abstract. We use the large N_f self consistency formalism to compute the $O(1/N_f)$ critical exponent corresponding to the renormalization of the flavour non-singlet twist two Wilson operators which arise in the operator product expansion of currents in deep inelastic processes. Expanding the d -dimensional expression in powers of $\epsilon = (4 - d)/2$ the coefficients of ϵ agree with the known two loop structure of the corresponding renormalization group function and we deduce analytic expressions for all moments, n , at three and higher orders in perturbation theory in the $\overline{\text{MS}}$ scheme at $O(1/N_f)$.

The quantum field theory describing the dynamics of strongly interacting particles is quantum chromodynamics, (QCD), which is an asymptotically free gauge theory, [1, 2]. In other words at increasingly higher energies the theory behaves more and more like a free theory and therefore the machinery of perturbative quantum field theory can be applied to gain an accurate understanding of physical processes. For example, the phenomenology of deep inelastic scattering can be explored and the one and two loop QCD corrections to parton model predictions can be computed which are found to be in good agreement with current experiments. (See, for example, [3].) With the increase in energy that will soon become available at LHC there would now appear to be a specific need to begin to examine subsequent three loop corrections to these processes, since the error bars on experimental results will be refined substantially.

One of the central ingredients of the scattering formalism in gauge theories is the operator product expansion, (OPE), of electromagnetic and other currents, [4], which are composite operators of the fields of the standard model and QCD lagrangians. The decomposition of the Wilson OPE is in terms of an infinite sum of composite operators which are either flavour non-singlet or singlet, [4]. However, these operators require renormalization. In other words, radiative corrections generate a non-zero anomalous dimension, $\gamma_{\text{NS}}^{(n)}(g)$, for the n th moment of, say, each non-singlet operator, where g is the coupling constant which will be defined later. Knowledge of $\gamma_{\text{NS}}^{(n)}(g)$ and the anomalous dimensions of the singlet operators are essential for determining, for example, corrections to sum rules. Although the Wilson coefficients are also required for this, where these are the coefficients which appear with each operator of the OPE, they depend on the specific process involved, unlike the anomalous dimensions, and therefore require separate treatment, [3]. Like the functions of the renormalization group equation, (RGE), $\gamma_{\text{NS}}^{(n)}(g)$ are calculated order by order in perturbation theory. Specifically one renormalizes the operator as a zero momentum insertion in some Green's function. As a result the one loop structure of non-singlet operators, $\gamma_{\text{NS}}^{(n)}(g)$, was given in [4] whilst the two loop analysis was carried out in [5-7]. (Two loop results for the singlet functions have been deduced in [4,8-11].) So it is the perturbative structure of these functions which must be determined at three loops in order to refine the theoretical understanding of physical processes.

Such calculations, however, are formidable in their computational complexity due to the huge number of Feynman diagrams to be analysed. In-

deed the two loop results were performed using computer algebra packages. Therefore it is also important to have an independent way of calculating information on the perturbation series which agrees with known calculations and, equally, provides additional non-trivial information at much higher orders. In this letter we introduce such a technique for computing the anomalous dimensions of physical operators involved in the OPE of deep inelastic scattering, though for simplicity we will only concentrate on the non-singlet case to illustrate the power and beauty of the method. (As the Wilson coefficients are process dependent they are beyond the scope of this letter.) The formalism is based on the large N_f self consistency programme which was first introduced in [12, 13] for the $O(N)$ bosonic σ model. To deduce information on the perturbative structure of any function of the RGE, one computes the relevant critical exponent at the non-trivial zero, g_c , of the d -dimensional β -function of QCD. There the theory is finite and the propagators of the fields and Green's functions have a simple power law or conformal structure. The associated critical exponent is, by universality, a function of the spacetime dimension, d , and any internal parameters. For the present letter, these will be N_f , the number of quark flavours, and N_c the number of colours. The benefit of proceeding in this fashion is that the RGE simplifies because of $\beta(g_c) = 0$, $g_c \neq 0$, to the extent that the d -dimensional critical exponent is simply related to the corresponding RGE function evaluated at the critical coupling. (See, for example, [14].) Therefore, if one computes in some approximation, which will be large N_f here, then knowledge of the critical exponent in d -dimensions means calculating order by order in $1/N_f$ one can deduce the coefficients of the perturbative function away from criticality. At low orders of g the coefficients deduced from the exponent will agree with those calculated in the mass independent $\overline{\text{MS}}$ scheme which uses dimensional regularization. Clearly this will be a powerful way of proceeding since one will always be able to obtain information to all orders in g at each level in $1/N_f$. Indeed it is one aim of this letter to draw attention to this alternative method of calculating the perturbation series of useful physical quantities.

We recall for the interested reader that the earlier applications of the large N_f exponent programme to four dimensional gauge theories included the evaluation of the electron anomalous dimension at $O(1/N_f^2)$, [15], the QED β -function at $O(1/N_f)$, [16], and the electron mass anomalous dimension at $O(1/N_f^2)$, [17]. As the latter involved the computation of the exponent of the mass operator $\bar{\psi}\psi$ which is also composite, we have used [17]

as a foundation for the present work. More recently, the quark, gluon and ghost anomalous dimensions have been deduced at $O(1/N_f)$ in [18] which are in agreement with the previous three loop results of [1,19-22].

To proceed with the critical point approach we recall the form of the QCD lagrangian we use is

$$\begin{aligned}
L = & -\frac{(F_{\mu\nu}^a)^2}{4e^2} + i\bar{q}^{iI}\not{\partial}q^{iI} + A_\mu^a\bar{q}^{iI}T_{IJ}^a\gamma^\mu q^{iJ} - \frac{1}{e^2}f^{abc}\partial_\mu A_\nu^a A^{\mu b}A^{\nu c} \\
& - \frac{1}{4e^2}f^{abc}f^{ade}A_\mu^b A_\nu^c A^{\mu d}A^{\nu e} - \frac{1}{2\xi e^2}(\partial_\mu A^{\mu a})^2 \\
& - \partial^\mu \bar{c}^a \partial_\mu c^a + f^{abc}\partial_\mu \bar{c}^a c^b A^{\mu c}
\end{aligned} \tag{1}$$

where q^{iI} is the quark field, $1 \leq i \leq N_f$, $1 \leq I, J \leq N_c$, A_μ^a is the gluon field with $1 \leq a \leq N_c^2 - 1$, $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$, c^a and \bar{c}^a are the ghost fields, ξ is the covariant gauge parameter, T_{IJ}^a are the generators of $SU(N_c)$ which has structure constants f^{abc} and e is the coupling constant. The location of g_c in the neighbourhood of which we will base our analysis is given by $\beta(g_c) = 0$, where to one loop in d -dimensions, [1],

$$\beta(g) = (d-4)g + \left[\frac{2}{3}T(R)N_f - \frac{11}{6}C_2(G) \right] g^2 + O(g^3) \tag{2}$$

where the dimensionless coupling is $g = (e/2\pi)^2$, in the notation of [23], and

$$\text{Tr}(T^a T^b) = T(R)\delta^{ab}, \quad T^a T^a = C_2(R)I, \quad f^{acd}f^{bcd} = C_2(G)\delta^{ab} \tag{3}$$

are the Casimirs of a general classical Lie group. For $SU(N_c)$, $T(R) = \frac{1}{2}$, $C_2(R) = (N_c^2 - 1)/2N_c$ and $C_2(G) = N_c$. Therefore from (2)

$$g_c = \frac{3\epsilon}{T(R)N_f} + O\left(\frac{1}{N_f^2}\right) \tag{4}$$

where $d = 4 - 2\epsilon$ but it turns out that to deduce the coefficients for $\gamma_{\text{NS}}^{(n)}(g)$ from the exponent $\eta^{(n)} \equiv \gamma_{\text{NS}}^{(n)}(g_c)$ we calculate at $O(1/N_f)$ here, the $O(1/N_f^2)$ corrections of (4) are not required.

The Wilson operators whose critical exponent $\eta^{(n)}$ we will compute are the twist 2 flavour non-singlet operators, [4],

$$\mathcal{O}_{\text{NS},a}^{\mu_1 \dots \mu_n, \pm} = \frac{1}{2}i^{n-1}\mathcal{S}\bar{q}^I\gamma^{\mu_1}D^{\mu_2} \dots D^{\mu_n}T_{IJ}^a(1 \pm \gamma^5)q^J - \text{trace terms} \tag{5}$$

where $D^\mu = \partial^\mu + iT^a A^{\mu a}$ and \mathcal{S} denotes the symmetrization of the Lorentz indices. At leading order in large N_f it will turn out that the γ^5 term

will be completely passive and the same result obtained for either sign. So we will suppress the index \pm for the moment. To determine the anomalous dimensions one first of all removes the symmetrization and trace terms by introducing a null vector Δ^μ with $\Delta^2 = 0$ and multiplies $\mathcal{O}_{\text{NS}}^{\mu_1 \dots \mu_n}$ by $\Delta_{\mu_1} \dots \Delta_{\mu_n}$ to simplify the number of insertions to be made in the Green's function, $\langle q \mathcal{O}_{\text{NS}} \bar{q} \rangle$, [4]. Next as the anomalous dimension $\gamma_{\text{NS}}^{(n)}(g)$ is gauge independent one chooses to compute in the Feynman gauge to reduce the algebra involved in the calculation. As a consequence of both these steps the insertion of the operator (5) requires the development of new Feynman rules. These have been derived in [4] for the one loop case and in [5] for the 2-loop calculations. We will use the former rules for the large N_f computation since at leading order the only graphs which contribute are those of fig. 1, where the dotted line corresponds to quark fields. The circle with cross denotes the zero momentum insertion of \mathcal{O}_{NS} . (In the singlet calculation there would be additionally two 2-loop graphs with a closed quark loop, [8], contributing at this order in $1/N_f$, but these are zero in the non-singlet case.)

In the perturbative calculation one computes each graph of fig. 1 with the conventional quark and Feynman gauge gluon propagators, \not{k}/k^2 and $\eta_{\mu\nu}/k^2$ respectively, in dimensional regularization and absorbs the simple poles in ϵ minimally into the appropriate renormalization constant. In the critical point approach one uses an alternative strategy. Near g_c , (4), the propagators of the fields obey asymptotic scaling and therefore satisfy a simple power law structure consistent with Lorentz and conformal symmetry. The general forms have been used extensively before, [15, 18], and in momentum space we recall that they are, in the Feynman gauge,

$$q(k) \sim \frac{\tilde{A} \not{k}}{(k^2)^{\mu-\alpha}} \quad , \quad A_{\nu\sigma}(k) \sim \frac{\tilde{B} \eta_{\nu\sigma}}{(k^2)^{\mu-\beta}} \quad (6)$$

where \tilde{A} and \tilde{B} are momentum independent amplitudes, $\mu = \frac{1}{2}d$ and the critical exponents α and β are defined as

$$\alpha = \mu - 1 + \frac{1}{2}\eta \quad , \quad \beta = 1 - \eta - \chi \quad (7)$$

Here the canonical dimension of each field is deduced from a dimensional analysis of the kinetic terms and interactions of (1) and the anomalous terms η and χ are respectively the quark wave function renormalization exponent and the qqg vertex anomalous dimension. (At leading order in $1/N_f$ there are no graphs involving ghosts, [4].) The presence of the non-zero anomalous

dimensions means that when considering subsequent orders in $1/N_f$ only those graphs where the quark and gluon propagators are not dressed are considered. From [17, 18] we recall that in the Feynman gauge

$$\eta_1 = \frac{2(\mu-1)^2 C_2(R) \eta_1^O}{(2\mu-1)(\mu-2)T(R)} \quad (8)$$

where $\eta = \sum_{i=1}^{\infty} \eta_i/N_f^i$ and $\eta_1^O = (2\mu-1)(\mu-2)\Gamma(2\mu)/[4\Gamma^2(\mu)\Gamma(\mu+1)\Gamma(2-\mu)]$. Also, the combination $z = \tilde{A}^2 \tilde{B}$ is required and [17, 18]

$$z_1 = \frac{\mu\Gamma(\mu)\eta_1^O}{2(\mu-2)(2\mu-1)T(R)} \quad (9)$$

We now recall the method to compute the anomalous dimensions of composite operators with the critical propagators (6). First, one substitutes the lines of fig. 1 with (6) and calculates the integral. However, one would discover that the integral is divergent and thus requires regularization. This is achieved by shifting the gluon exponent by an infinitesimal quantity δ , $\beta \rightarrow \beta - \delta$, [13, 15, 18]. (In previous work, we used the symbol Δ for the regularizing parameter but to avoid confusion with Δ_μ we use δ here.) When one computes with non-zero δ now, each graph of fig. 1 will have the following formal structure at $O(1/N_f)$ after expanding in powers of δ , [24, 17],

$$\frac{P}{\delta} + Q + R \ln p^2 + O(\delta) \quad (10)$$

where P , Q and R depend on μ and N_c , and p is the external momentum flowing through the quark fields. The simple pole is absorbed minimally by a conventional (critical point) renormalization, [24]. This leaves a term involving R which would violate scaling symmetry in the limit as one approaches criticality, $p^2 \rightarrow \infty$. To avoid this one notes that by general arguments the δ -finite Green's function must resum to the structure $(p^2)^{\gamma_O^{(n)}(g_c)/2}$, [24], where $\gamma_O^{(n)}(g_c)$ is related to the exponent we are interested in, $\eta_1^{(n)}$, through, [17],

$$\eta^{(n)} = \eta + \gamma_O^{(n)}(g_c) \quad (11)$$

and is what is obtained from the insertion of (5) in the graphs of fig. 1. Therefore, by isolating and then summing the contributions from the $\ln p^2$ terms of each graph of fig. 1, we can deduce $\eta_1^{(n)}$ via (8) and (11).

Using (6) and the Feynman rules of [4], we find that the first graph of fig. 1 contributes

$$- \frac{2\mu(\mu-1)^3 C_2(R) \eta_1^0}{(\mu-2)(2\mu-1)(\mu+n-1)(\mu+n-2)T(R)} \quad (12)$$

to $\gamma_{\mathcal{O}}^{(n)}(g_c)$, whilst the second (together with its mirror image) gives

$$\frac{4\mu(\mu-1)C_2(R)\eta_1^0}{(\mu-2)(2\mu-1)T(R)} \sum_{l=2}^n \frac{1}{(\mu+l-2)} \quad (13)$$

Therefore, from (8) and (11) we have

$$\begin{aligned} \eta_1^{(n)} &= \frac{2C_2(R)(\mu-1)^2 \eta_1^0}{(2\mu-1)(\mu-2)T(R)} \left[\frac{(n-1)(2\mu+n-2)}{(\mu+n-1)(\mu+n-2)} \right. \\ &\quad \left. + \frac{2\mu}{(\mu-1)} \sum_{l=1}^n \frac{1}{(\mu+l-2)} - \frac{2\mu}{(\mu-1)^2} \right] \end{aligned} \quad (14)$$

which is the main result of this letter¹.

There are several checks we can make on (14). First, in the case $n = 1$, the anomalous dimension $\gamma_{\text{NS}}^{(1)}(g)$ is zero, [4], since then \mathcal{O}_{NS} corresponds to a conserved current. It is easy to verify from (14) that $\eta_1^{(1)} = 0$ in agreement with this general result. Second, we can expand $\eta_1^{(n)}$ in powers of ϵ in $d = 4 - 2\epsilon$ dimensions and compare with the 2-loop result of [4-7] ie

$$\begin{aligned} \gamma_{\text{NS}}^{(n),\pm}(g) &= C_2(R)g \left[2S_1(n) - \frac{3}{2} - \frac{1}{n(n+1)} \right] \\ &\quad + \left[\frac{C_2(R)T(R)N_f}{9} (6S_2(n) - 10S_1(n)) \right. \\ &\quad \left. + \frac{3}{4} + \frac{(11n^2 + 5n - 3)}{n^2(n+1)^2} \right] g^2 \end{aligned} \quad (15)$$

where $S_l(n) = \sum_{i=1}^n 1/i^l$ and the coefficients $b^{\pm}(n)$ have been given in [5-7] but are $O(1/N_f^2)$ and their explicit form is not needed for the present point. Substituting the critical coupling g_c in (15) from (4) and comparing with

¹Although the factor of $(\mu-2)$ in the denominator of (14) suggests that $\eta_1^{(n)}$ behaves as $1/\epsilon$ as $\epsilon \rightarrow 0$ when $\mu = 2 - \epsilon$, in fact $\eta_1^0 = O(\epsilon^2)$ which means that $\eta_1^{(n)}$ and $\gamma_{\text{NS}}^{(n)}(g_c)$ are both $O(\epsilon)$.

the ϵ expansion of (14) it is easy to verify agreement with the two $O(1/N_f)$ coefficients of (15). This is a highly non-trivial check and establishes the correctness of (14) since we considered only the leading order $1/N_f$ graphs which are one loop. In other words the conformal ansätze and formalism correctly reproduce the large N_f bubble sum contributions at higher order. More importantly having verified its correctness we can now deduce subsequent terms in the perturbation series albeit at leading order in $1/N_f$. For example, if we set

$$\gamma_{\text{NS}}^{(n)}(g) = a_1 C_2(R)g + \sum_{i=2}^{\infty} a_i C_2(R) [T(R)N_f]^{i-1} g^i \quad (16)$$

for the leading order $1/N_f$ part of the function, where a_1 and a_2 can be read off from (15), then (14) implies that

$$a_3 = \frac{2}{9}S_3(n) - \frac{10}{27}S_2(n) - \frac{2}{27}S_1(n) + \frac{17}{72} - \frac{[12n^4 + 2n^3 - 12n^2 - 2n + 3]}{27n^3(n+1)^3} \quad (17)$$

$$a_4 = \frac{2}{27}S_4(n) - \frac{10}{81}S_3(n) - \frac{2}{81}S_2(n) - \frac{2}{81}S_1(n) + \left[\frac{4}{27}S_1(n) - \frac{2}{27n(n+1)} - \frac{1}{9} \right] \zeta(3) + \frac{131}{1296} + \frac{[4n^6 - 12n^5 - 15n^4 + 10n^3 + 14n^2 - n - 3]}{81n^4(n+1)^4} \quad (18)$$

$$a_5 = \frac{2}{243}[3S_5(n) - 5S_4(n) - S_3(n) - S_2(n) - S_1(n)] + \frac{\zeta(4)}{54} \left[4S_1(n) - 3 - \frac{2}{n(n+1)} \right] + \frac{323}{7776} + \frac{\zeta(3)}{486} \left[24S_2(n) - 40S_1(n) + 3 + \frac{4[11n^2 + 5n - 3]}{n^2(n+1)^2} \right] + \frac{[8n^7 - 8n^6 - 28n^5 - 5n^4 + 24n^3 + 13n^2 - 4n - 3]}{243n^5(n+1)^5} \quad (19)$$

are the subsequent new coefficients, where $\zeta(q)$ is the Riemann zeta function. Of course, a_3 will be an important check for the explicit 3-loop $\overline{\text{MS}}$ calculation of $\gamma_{\text{NS}}^{(n)}(g)$ which is currently being calculated for various moments, [25]. To aid comparison of the $O(1/N_f)$ part of the full three loop result of [25], we have evaluated a_3 for even n , $2 \leq n \leq 22$, and collected the

results in table 1. In [25], the coefficient of the $n = 8$ case was given and it is very satisfying to record that there is exact agreement with the corresponding entry of our table, which reinforces our confidence in the correctness of the exponent (14), (after allowing for the different convention in defining the coupling constant in this paper and an overall factor of 2 between the definition of the renormalization group functions). The remaining values of table 1 will occur in the explicit evaluation of $\gamma_{\text{NS}}^{(n)}(g)$ in perturbation theory for other n .

Finally, with (14) the x -behaviour of the $O(1/N_f)$ part of the non-singlet splitting function, $P_{\text{NS}}(g, x)$, of the Altarelli-Parisi equations can now be examined, where x is the Bjorken scaling variable. This will be useful in the extension of the earlier 2-loop work of [11, 26]. It is defined as the Mellin transform of the anomalous dimension,

$$\int_0^1 dx x^{n-1} P_{\text{NS}}(g, x) = -\frac{1}{4} \gamma_{\text{NS}}^{(n)}(g) \quad (20)$$

where we use the conventions of [27].

We conclude by noting that we have given an alternative way of computing anomalous dimensions of physical operators in QCD, which agrees with low order results, and also provided the means to deduce the higher order structure. It ought now to be possible to develop this large N_f formalism in two directions. First, the $O(1/N_f^2)$ corrections can, in principle, be deduced. However, one would first need to know η_2 , the quark anomalous dimension, which has yet to be computed. Secondly, it may be possible to determine the singlet operator anomalous dimensions. Although these mix under renormalization that is not a significant problem since matrices of exponents have been deduced in other contexts within the critical point programme, [24].

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n	a_3
2	$-\frac{28}{243}$
4	$-\frac{384277}{1944000}$
6	$-\frac{80347571}{333396000}$
8	$-\frac{38920977797}{144027072000}$
10	$-\frac{27995901056887}{95850016416000}$
12	$-\frac{65155853387858071}{210582486065952000}$
14	$-\frac{68167166257767019}{210582486065952000}$
16	$-\frac{5559466349834573157251}{16553468064672354816000}$
18	$-\frac{19664013779117250232266617}{56770118727793840841472000}$
20	$-\frac{6730392290450520870012467}{18923372909264613613824000}$
22	$-\frac{16759806821032136669044226177}{46048135637404510767879321600}$

Table 1: Values of a_3 for various n .

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Figure Captions.

Fig. 1. Leading order graphs for $\eta_1^{(n)}$.

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